Twist Three Generalized Parton Distributions For Orbital Angular Momentum

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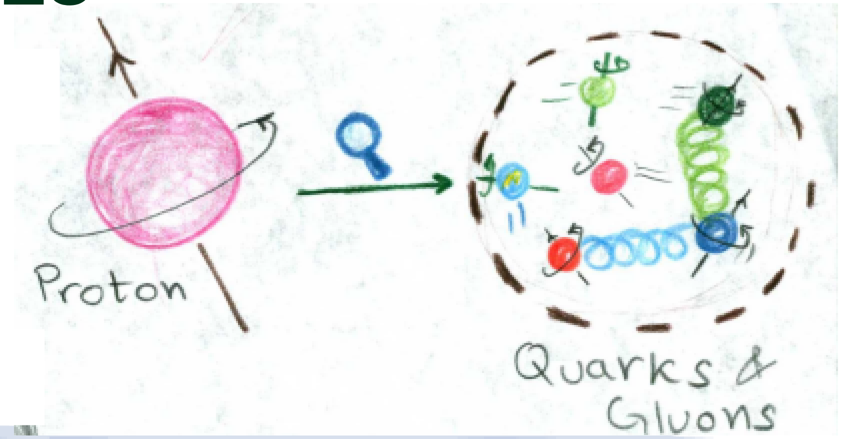
University of Virginia

HUGS, 2015

Simonetta Liuti,
Aurore Courtoy,
Michael Engelhardt



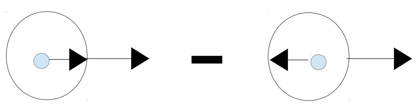
Proton Spin Crisis

- Quark and gluon spin contribution is smaller than ½
- Need to look at other Sources of spin → Orbital Angular Momentum

$1+\gamma^5$ ← Helicity Projection Operator

$$\frac{1}{2M}\int_{-\infty}^{\infty}\frac{d\lambda}{2\pi}e^{-i\lambda x}\left\langle P,S\left|\bar{\psi}\left(\frac{\lambda n}{2}\right)\gamma_\mu\gamma_5\psi\left(-\frac{\lambda n}{2}\right)\right|P,S\right\rangle = \Lambda g_1(x)p_\mu + g_T(x)S_{\perp_\mu}$$

$$\int_0^1 dx g_1^P(x) = \frac{1}{2}\left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right]$$



g_1

Measured by EMC experiment in 1980s to be only 33% of total !!

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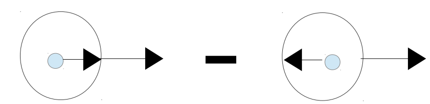

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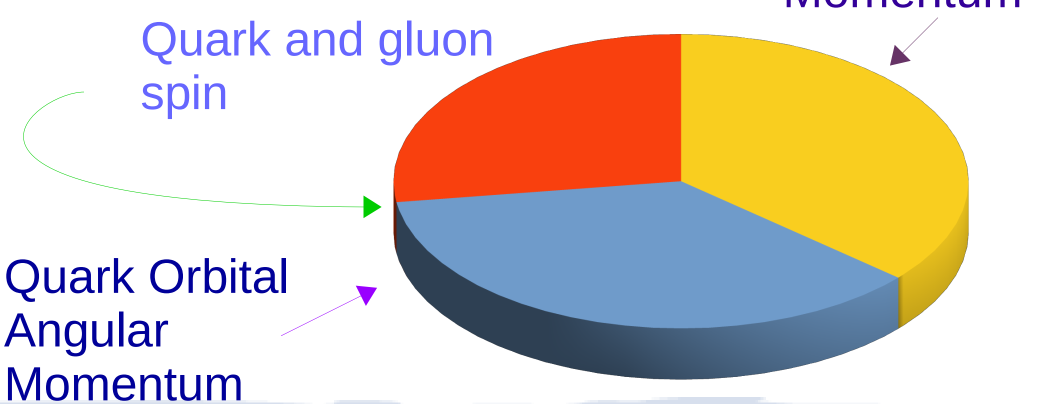

Jaffe Manohar and Ji Decompositions of Proton Spin

- Differ in their definition of Orbital Angular Momentum

$$\vec{\mathcal{L}}_{Ji} \to i\vec{r} \times \vec{D} \qquad \vec{L}_{JM} \to i\vec{r} \times \vec{\partial}$$

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$$\frac{1}{2} = J_q + J_g$$

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- Jaffe Manohar: Spin and OAM contributions are separate

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + L_q + \Delta G + L_g$$

Form Factors and PDFs

Elastic Scattering

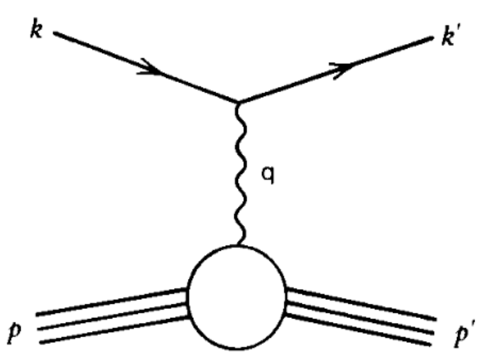
$$j^\mu = -e\,\bar{u}(k')\gamma^\mu u(k)\,e^{i(k'-k)\cdot x}$$

$$J^\mu = e\,\bar{u}(p')[\ ?\ ]u(p)\,e^{i(p'-p)\cdot x},$$

$$[\ ?\ ] = \left[F_1(q^2)\gamma^\mu + \frac{\kappa}{2M}F_2(q^2)\,i\sigma^{\mu\nu}q_\nu\right]$$

Electron Current
- Spin ½ point Particle
- No internal Structure

Unknown Proton Current
- Parametrized using form factors



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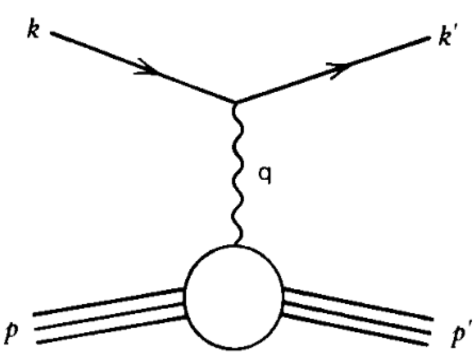
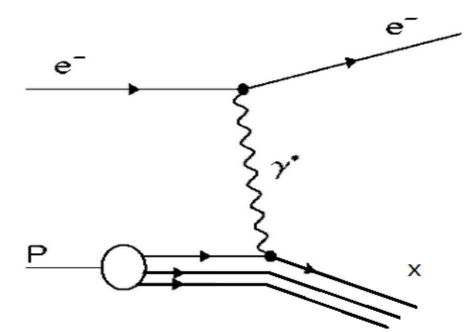


Elastic Scattering

Deep Inelastic Scattering

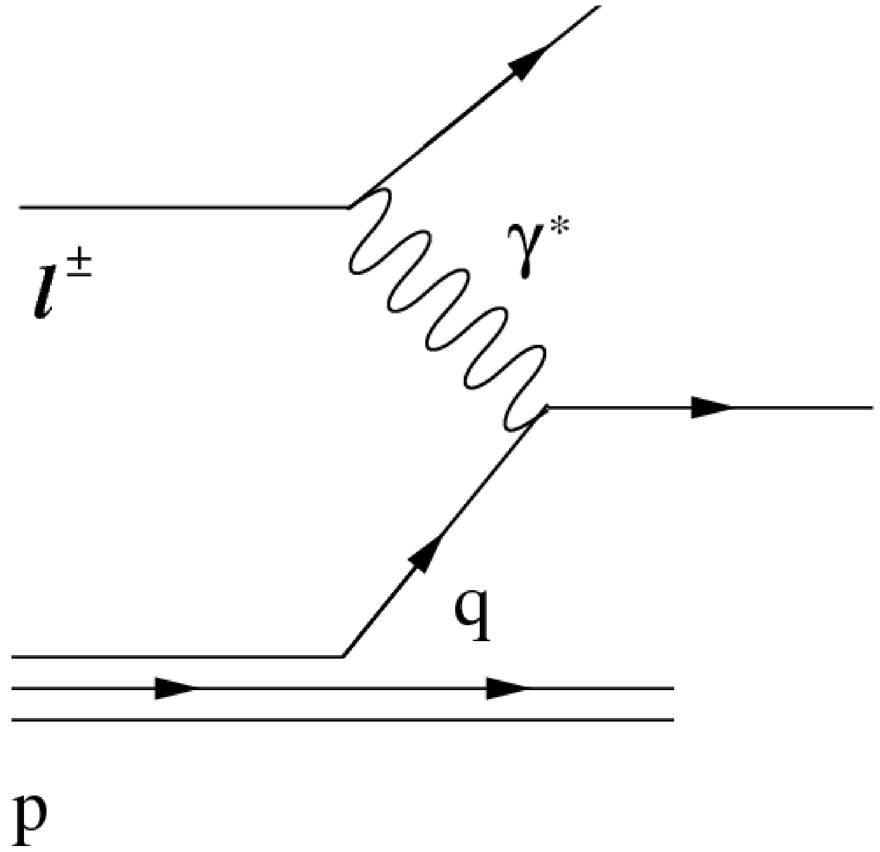
Express cross section in terms of Parton Distribution Functions

$$W_{\mu\nu} = \sum_i \sum_s \int d^4k\, f_s^i(p,k)\, w_{\mu\nu}^i$$



$$\int \frac{dz^-}{2\pi} e^{ixz^-} \langle p,S \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p,S\rangle_{z^+=z_T=0} = f_1(x)$$

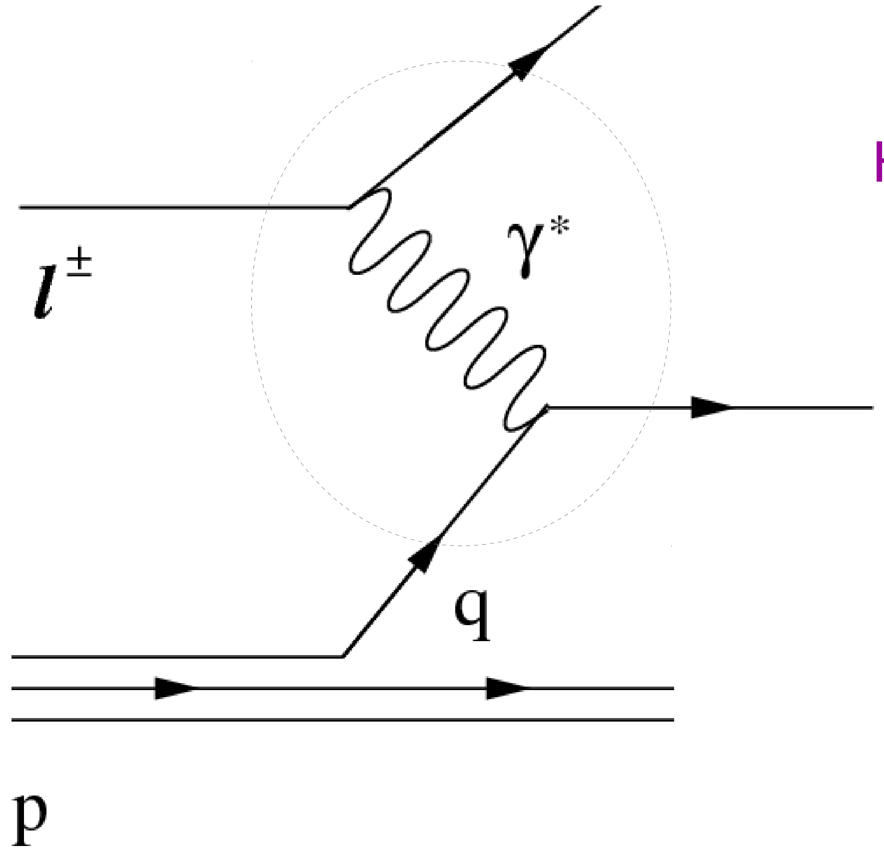
Hard and Soft Parts



Deep Inelastic Scattering

Fillipone et Ji, Adv Nucl Phys 26 (2001)

Hard and Soft Parts



Hard Interaction

Deep Inelastic Scattering

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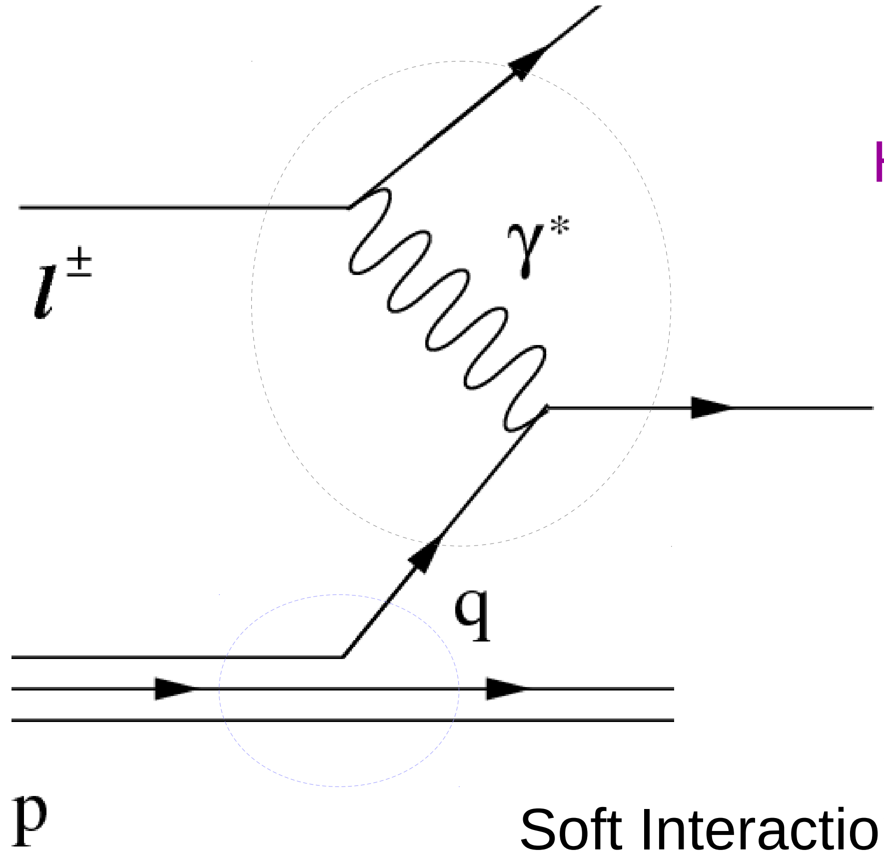

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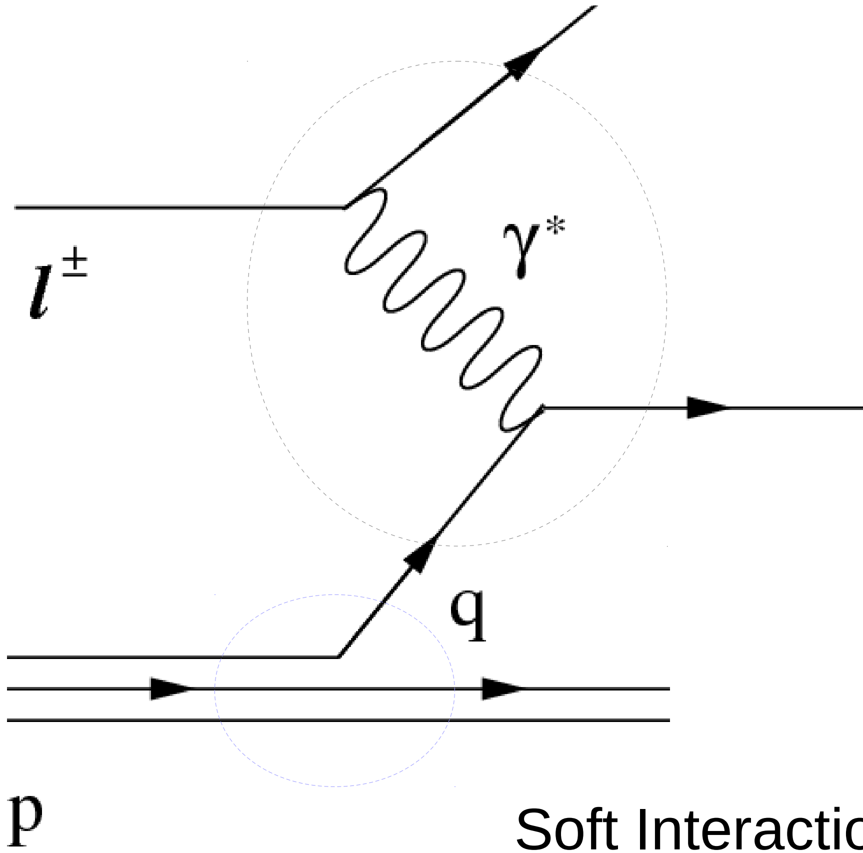

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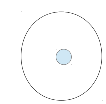
$$\int \frac{dz^-}{2\pi} e^{ik^+z^-} \langle p,S \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p,S\rangle_{z^+=z_T=0} = f_1(x)$$

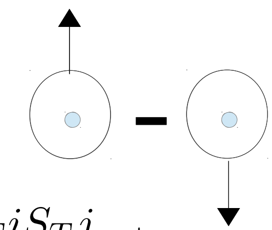
$$a^{\pm} = \frac{a^0 \pm a^3}{\sqrt{2}}$$

Fillipone et Ji, Adv Nucl Phys 26 (2001)

Transverse Momentum Distributions

- Transverse Momentum Distributions are like Parton Distribution Functions but, also include transverse momentum of quarks





$$\int \frac{dz_{-} d^2 z_T}{2\pi} e^{ixP^+ z^- - k_T . z_T} \langle p, S \mid \bar{\psi}(-z/2) \gamma^+ \psi(z/2) \mid p, S \rangle_{z^+=0} = f_1(x, k_T) - \frac{\epsilon^{ij} k_T i S_T j}{M} f_{1T}^{\perp}(x, k_T)$$

Boglione, Mulders Phys Rev D60 (1999)

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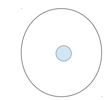

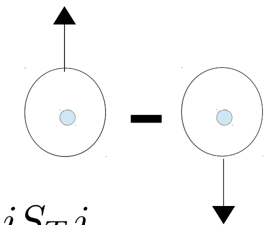

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$$\int d^2 k_T f_1(x, k_T) = f_1(x)$$

TMDs → Unintegrated PDFs

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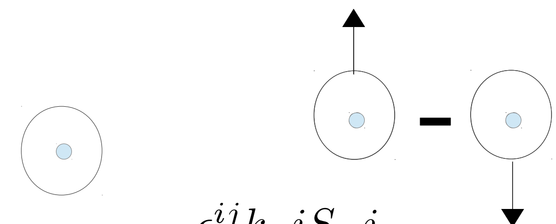

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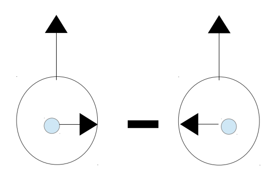
TMDs → Unintegrated PDFs

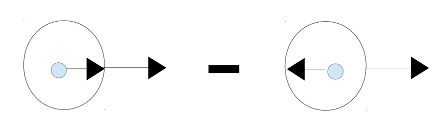
- Projection Operator : Access different properties of the partonic structure of nucleon with different projection operators

$$\Phi[\gamma^+\gamma^5] = g_{1L}(x, k_T^2)\lambda + g_{1T}(x, k_T^2)\frac{k_T . S_T}{M}$$

Boglione, Mulders Phys Rev D60 (1999)
Jakob, Mulders and Rodrigues, Nucl Phys A626 (1997)



g_{1T}



g_{1L}

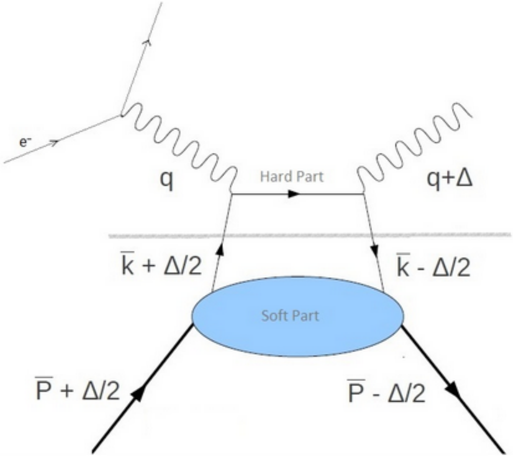
GPDs and GTMDs

- Generalized Parton Distributions : Off Forward PDFs

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', \Lambda' \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, \Lambda \rangle_{z^+=z_T=0} = \bar{U}(P', \Lambda')(\gamma^+ H(x,\xi,t) + \frac{i\sigma^{+\mu}\Delta_\mu}{2M} E(x,\xi,t))U(P,\Lambda)$$

$$\xi = \frac{\Delta^+}{P^+} \qquad t = \Delta^2 \qquad \Delta = P' - P$$

Xiangdong Ji, PRL 78.610,1997



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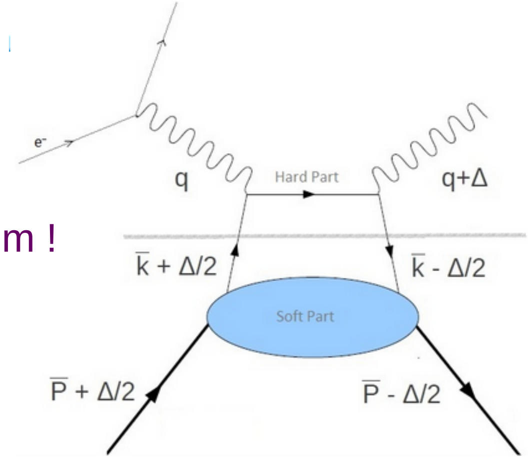
- Enter at amplitude level

$$\vec{J}_q = \int d^3x\, \psi^\dagger [\vec{\gamma}\gamma_5 + \vec{x} \times (-i\vec{D})]\psi$$

Ji Sum Rule : partonic angular momentum !

$$J_q = \frac{1}{2}\int_{-1}^{1} dx\, x(H_q(x,0,0) + E_q(x,0,0))$$



DVCS

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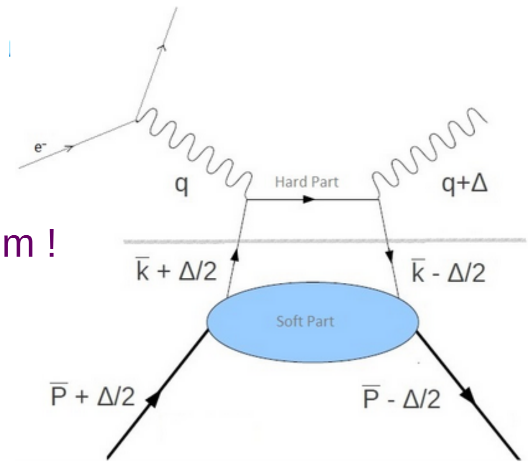

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- Generalised Transverse Momentum Distributions : Off forward TMDs

$$W^{\gamma^+}_{\Lambda,\Lambda'} = \frac{1}{2M}\bar{u}(p',\Lambda')[F_{11} + \frac{i\sigma^{i+}k^i_T}{\bar{p}_+}F_{12} + \frac{i\sigma^{i+}\Delta^i_T}{\bar{p}_+}F_{13} + \frac{i\sigma^{ij}k^i_T\Delta^j_T}{M^2}F_{14}]u(p,\Lambda)$$

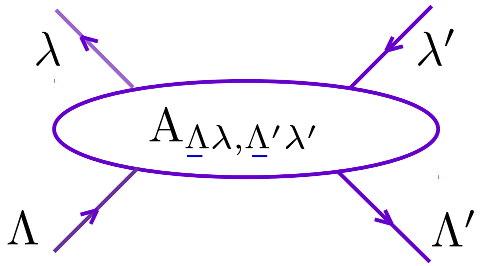
Jaffe Manohar OAM

Functions of $x, k_T^2, k_T.\Delta_T, \xi, t$

Meissner Metz and Schlegel, JHEP 0908 (2009)

Helicity Amplitudes

- Helicity structure of the quark quark correlator (soft part)



$$\gamma^+(1 \pm \gamma^5) \rightarrow A_{\Lambda,\pm,\Lambda,\pm}$$

Projects Helicity

$$\frac{1}{2}[\gamma^+(1+\gamma^5) + \gamma^+(1-\gamma^5)] = \gamma^+$$

$$\int \frac{dz_-}{2\pi} e^{ixP^+z^-} \langle p', + \mid \bar{\psi}(-z/2)\gamma^+\psi(z/2) \mid p, + \rangle_{z_T=z^+=0} = A_{++,++} + A_{+-,+-}$$

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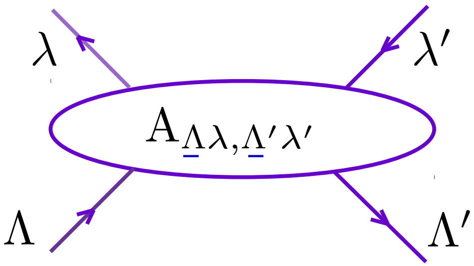

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$$\mathrm{H}(\mathrm{x},\xi,t) = A_{++,++} + A_{+-,+-} + A_{-+,-+} + A_{--,--}$$

$$\Delta_2 E(x,\xi,t) = A_{+_x+,+_x+} + A_{+_x-,+_x-} - A_{-_x+,-_x+} - A_{-_x-,-_x-}$$

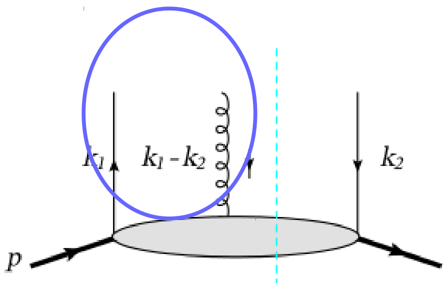
Helicity Flip treated as Transverse Polarization (transversity basis)

Twist 3

- The examples so far were at leading order

  What happens if we include a gluon on one side?

- Dynamical Twist → Suppressed by $\frac{1}{P^+}$



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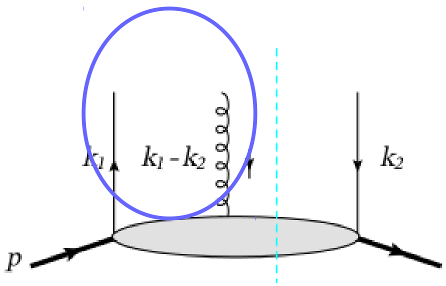

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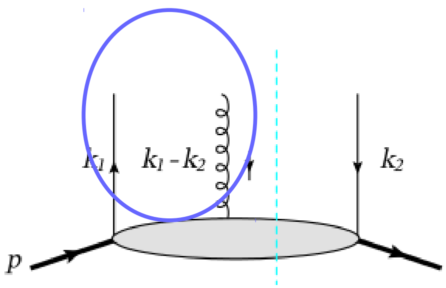

$$\langle p', \Lambda' \mid \bar{\psi}(-z/2)\Gamma\psi(z/2) \mid p, \Lambda\rangle$$

Projection operators determines whether distribution functions contribute at leading order or are suppressed (twist 3 or higher)

$$\phi^{[1]}(x, k_T^2) = \frac{M}{P^+} e(x, k_T^2)$$

$$W^{[\gamma_\perp^i]}(x, \zeta, t) = \bar{U}(P', S')[(H+E)\gamma_\perp^i + \frac{\Delta_\perp^i}{2M} G_1 + \gamma_\perp^i G_2 + \frac{\Delta_\perp^i \gamma^+}{P^+} G_3 + i\epsilon_\perp^{ij} \Delta_j^\perp \frac{\gamma^+ \gamma^5}{P^+} G_4] U(P, S)$$

Twist 3

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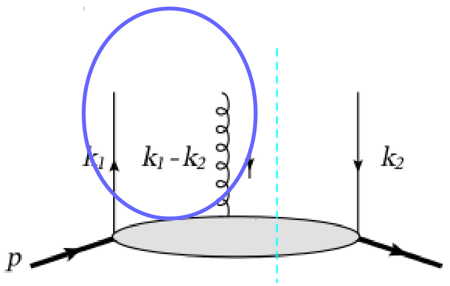
- Dynamical Twist → Suppressed by $\frac{1}{P^+}$
- Genuine Twist Three → Quark gluon quark correlator



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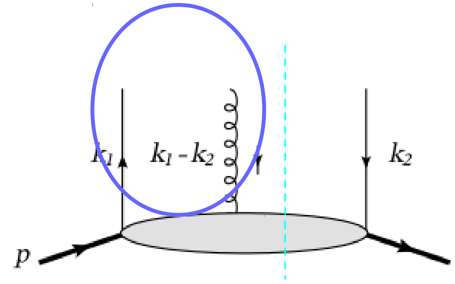
Genuine Twist 3

$$\boldsymbol{S}_T^i\,\tilde{g}_T(x) = \frac{1}{4xM}\int\frac{dx^-}{2\pi}e^{ixP^+x^-}\,\langle PS|\overline{\psi}(0)gA_{Tj}(0)[g_T^{ij}\gamma^+\gamma_5 + i\epsilon_T^{ij}\gamma^+]\psi(x^-)|PS\rangle + \text{h.c.},$$

$$\int\frac{dz^-}{2\pi}e^{ik^+z^-}\langle p,S\mid\bar{\psi}(-z/2)\gamma_T^i\gamma_5\psi(z/2)\mid p,S\rangle_{z^+=z_T=0} = \frac{M}{P^+}S_T^i g_T$$



$$g_T(x) = \int d^2k_T\frac{k_T^2}{2M^2}\frac{g_{1T}(x,k_T^2)}{x} + \frac{m}{M}\frac{h_1(x)}{x} + \tilde{g_T}(x)$$

Why is Twist 3 interesting ?

- We are getting access to phenomenon that involve three particle interactions
- Orbital Angular Momentum : nucleon spin, confinement

  OAM (Ji) → Twist 3 GPD G_2
- Color force d_2

Orbital Angular Momentum

- Polyakov Sum Rule → Twist three GPD G_2 gives partonic Orbital Angular Momentum (Ji)

$$-\int_{-1}^{1} dx\, x G_2(x,0,0) = \frac{1}{2}\left[\int_{-1}^{1} \tilde{H}(x,0,0) - \int_{-1}^{1} dx\, x(H(x,0,0)+E(x,0,0))\right]$$

Spin Contribution

Total Angular Momentum → J

Kiptily, Polyakov Eur Phys J C 37 (2004); Hatta and Yoshida, JHEP (1210), 2012

- Measuring G_2 → Connection to an observable

Courtoy, Liuti, Goldstein, Gonzalez, Rajan Phys Lett B731(2014)

- Jaffe Manohar OAM → Twist 2 GTMD F_{14}

Burkhardt Cottingham Sum Rule

$$\int \frac{d^4z}{(2\pi)^4} e^{ik.z} \langle p,S \mid \bar{\psi}(0)(iD(0)-m)i\sigma^{i+}\gamma_5\psi(z/2) \mid p,S\rangle = 0$$

$$\int \frac{d^4z}{(2\pi)^4} e^{ik.z} \langle p,S \mid \bar{\psi}(0)(k+gA(0)-m)i\sigma^{i+}\gamma_5\psi(z/2) \mid p,S\rangle = 0$$

$$k^+\Phi^{[\gamma^i\gamma^5]} - ik^+\epsilon^{ij}\Phi^{[\gamma^j]} - k^i\Phi^{[\gamma^+\gamma^5]} - ik^j\epsilon^{ij}\Phi^{[\gamma^+]} = 0$$

$$g_T(x) = \int d^2k_T \frac{k_T^2}{M^2} g_{1T}(x,k_T) + \tilde{g_T}(x)$$

Dynamic Tw 3

K_T squared moment in Tw 2

Genuine Tw 3 : Quark gluon quark correlator

What is this sum rule about? What does it say ? → Signifies that there is a twist 2 part and a twist 3 part

$$g_T(x) = g_1(x) + \frac{d}{dx} g_{1T}^{(1)}$$

$$M^3\Phi = a_1 + a_2\frac{\not{P}}{M} + a_3\frac{\not{k}}{M} + a_4\gamma_5\not{S} + a_7\frac{k.S}{M^2}\gamma_5\not{P} + a_8\frac{k.S}{M^2}\gamma_5\not{k} +$$
$$A_{12}\frac{\epsilon_{\mu\nu\rho\sigma}\gamma^\mu P^\nu k^\rho S^\sigma}{M^2} + a_5\frac{\gamma_5[\not{P},\not{S}]}{M} + a_6\frac{\gamma_5[\not{k},\not{S}]}{M} + a_9\frac{k.S\gamma_5[\not{P},\not{k}]}{M^3}$$

Unintegrated correlator

Mulders and Tangerman

$$g_T = g_1 + g_2$$

$$\int dx g_2(x) = 0$$

The color force : d_2

- d_2 → measure of color electric and magnetic force on quarks ; third moment of genuine twist 3 part of distributions

$$g_2(x) = g_2^{WW}(x) + \bar{g}_2(x)$$

$$2MP^+P^+S^x d_2^q = g\langle P,S|\bar{q}(0)\gamma^+ G^{+y}(0)q(0)|P,S\rangle$$

Burkardt PRD88 (2013)

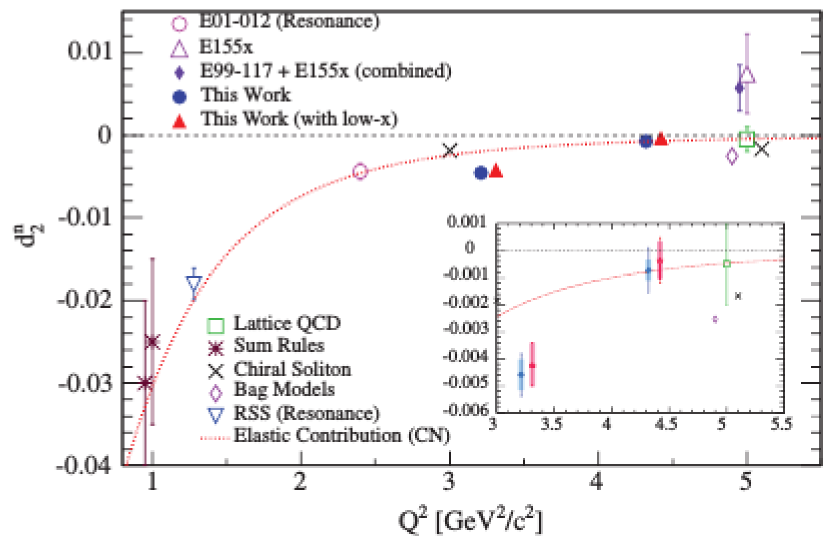
$$\int dx x^2 \bar{g}_2(x) = d_2$$

$$\int_{-1}^{1} dx\, x^2\, G_1^{tw3}(x,\xi) = 0,$$

$$\int_{-1}^{1} dx\, x^2\, G_2^{tw3}(x,\xi) = -\frac{2}{3}(1-\xi^2)\, d^{(2)},$$

$$\int_{-1}^{1} dx\, x^2\, G_3^{tw3}(x,\xi) = \frac{1}{3}\xi\, d^{(2)},$$

$$\int_{-1}^{1} dx\, x^2\, G_4^{tw3}(x,\xi) = \frac{1}{3} d^{(2)}.$$



Posik et al, PRL 111 (2013)

Connecting Jaffe and Ji OAM

Using Equation of Motion to get to OAM

$$\int \frac{d^4z}{(2\pi)^4} e^{ik.z} \langle p', S' \mid \bar{\psi}(0)(i\not{D}(0) - m) i\sigma^{i+}\gamma_5 \psi(z/2) \mid p, S\rangle = 0$$

$$\int \frac{d^4z}{(2\pi)^4} e^{ik.z} \langle p', S' \mid \bar{\psi}(0)(\not{k} + gA(0) - m) i\sigma^{i+}\gamma_5 \psi(z/2) \mid p, S\rangle = 0$$

Off forward correlator

$$k^+\Phi^{[\gamma^i\gamma^5]} - ik^+\epsilon^{ij}\Phi^{[\gamma^j]} - k^i\Phi^{[\gamma^+\gamma^5]} - ik^j\epsilon^{ij}\Phi^{[\gamma^+]} = 0$$

$$x\left(\frac{k_T.\Delta_T}{\Delta_T^2}G_{27} + G_{28}\right) + x\left(\frac{k_T.\Delta_T}{\Delta_T^2}F_{27} + F_{28}\right) = \frac{k_T^2}{M^2}\sin^2\phi F_{14} + \frac{k_T.\Delta_T}{\Delta_T^2}G_{14}$$

The correlator will now expand into GTMDs

$$2x\int d^k_T\left(\frac{k_T.\Delta_T}{\Delta_T^2}G_{27} + G_{28}\right) + 2x\int d^2k_T\frac{k_T.\Delta_T}{\Delta_T^2}F_{27} + F_{28} = 2\int d^2k_T\frac{k_T^2}{M^2}\sin^2\phi F_{14} + \frac{k_T.\Delta_T}{\Delta_T^2}G_{14}$$

Integrate over K_T

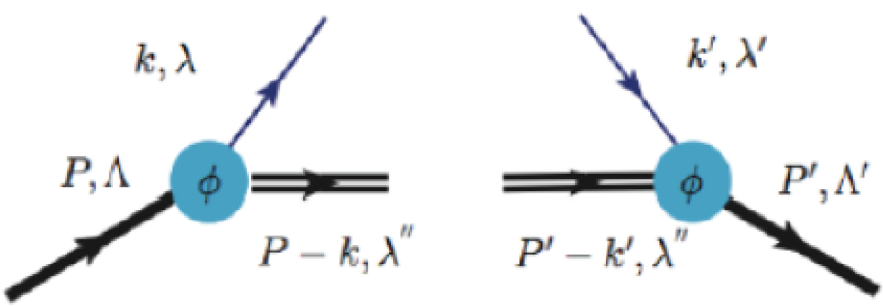
$$-xG_2 = 2\int d^2k_T\left(\frac{k_T^2}{M^2}\sin^2\phi F_{14} - x\left(\frac{k_T.\Delta_T}{\Delta_T^2}G_{27} + G_{28}\right) + d^2k\frac{k_T.\Delta_T}{\Delta_T^2}G_{14}\right)$$

Twist 3 GPD

K_T squared moment of Twist 2 GTMD

Diquark Model Calculation : Using Helicity Amplitudes

- The proton splits into a quark and a diquark structure. While the active quark interacts with the photon, the diquark acts as the 'spectator'



$$\phi_{\Lambda\lambda}(k,P) = \Gamma(k)\frac{\bar{u}(k,\lambda)U(P,\Lambda)}{k^2 - m^2}$$

$$\phi_{\Lambda'\lambda'}(k',P') = \Gamma(k')\frac{\bar{u}(k',\lambda')U(P',\Lambda')}{k'^2 - m^2}$$

$$A_{\Lambda,\lambda,\Lambda',\lambda'} = \phi^*_{\Lambda'\lambda'}(k',P')\phi_{\Lambda\lambda}(k,P)$$

Goldstein, Gonzalez, Liuti
PRD 84 (2011)

Calculating F_14

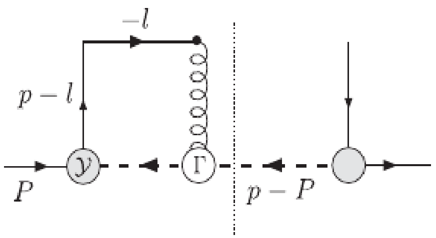
$$A_{++,++} + A_{+-,+-} - A_{-+,-+} - A_{--,--}$$

Unpolarized Quark in a longitudinally polarized proton

- Ji → Straight Gauge link

$$\mathcal{N}\frac{M^2}{x(k^2 - m_\Lambda^2)^2((k-\Delta)^2 - m_\Lambda^2)^2}$$

- Jaffe Manohar → Staple Link



$$\mathcal{N}\frac{M^2(1-x)}{x(k^2 - m_\Lambda^2)^2}\int\frac{d^2 l_T}{(2\pi)^2}\frac{(1+\frac{l_T.k_T}{l_T^2})}{((l_T - k_T)^2 - M^2(x) - 2x l_T^2)^2}$$

Bacchetta, Conti, Radici (2008)

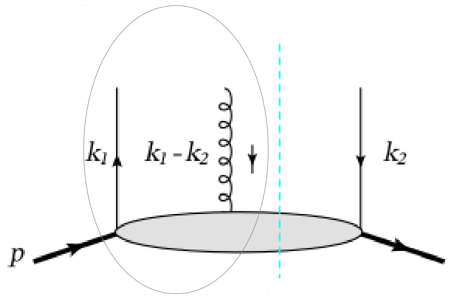
- The diference is the torque

$$L_q^{JM} - L_q^{Ji} = \int\frac{d^2 z_T dz^-}{(2\pi)^3}\langle P',\Lambda'|\bar{\psi}(z)\gamma^+(-g)\int_{z^-}^{\infty} dy^- U\left[z_1 G^{+1}(y^-) - z_2 G^{+2}(y^-)\right]U\psi(z)|P,\Lambda\rangle\Big|_{z^+=0}$$

Burkardt (2013)

Calculating Twist Three Amplitudes: Diquark Spectator Model

- The bad component is a quark gluon combination with spin opposite to that of the quark



$$\phi_{\Lambda,\lambda} = \Gamma(k)\frac{\bar{u}(k,\lambda)U(P,\Lambda)}{k^2 - m^2}$$

$$\phi^{tw3}_{\Lambda,\lambda^*} = \Gamma(k)\frac{\bar{u}(k,\lambda^*)U(P,\Lambda)}{k^2 - m^2}$$

$$A_{\Lambda,\lambda,\Lambda',\lambda'} = \phi^*_{\Lambda'\lambda'}(k',P')\phi_{\Lambda\lambda}(k,P)$$

Goldstein, Gonzalez, Liuti
PRD 84 (2011)

$$A^{tw3}_{\Lambda\pm,\Lambda\pm} = \langle p',\Lambda \mid \bar{\psi}(-z/2)(\gamma^1 \pm i\gamma^2)(1 \pm \gamma^5)\psi(z/2) \mid p,\Lambda\rangle$$

Courtoy, Liuti, Goldstein, Gonzalez, Rajan Phys Lett B731(2014)

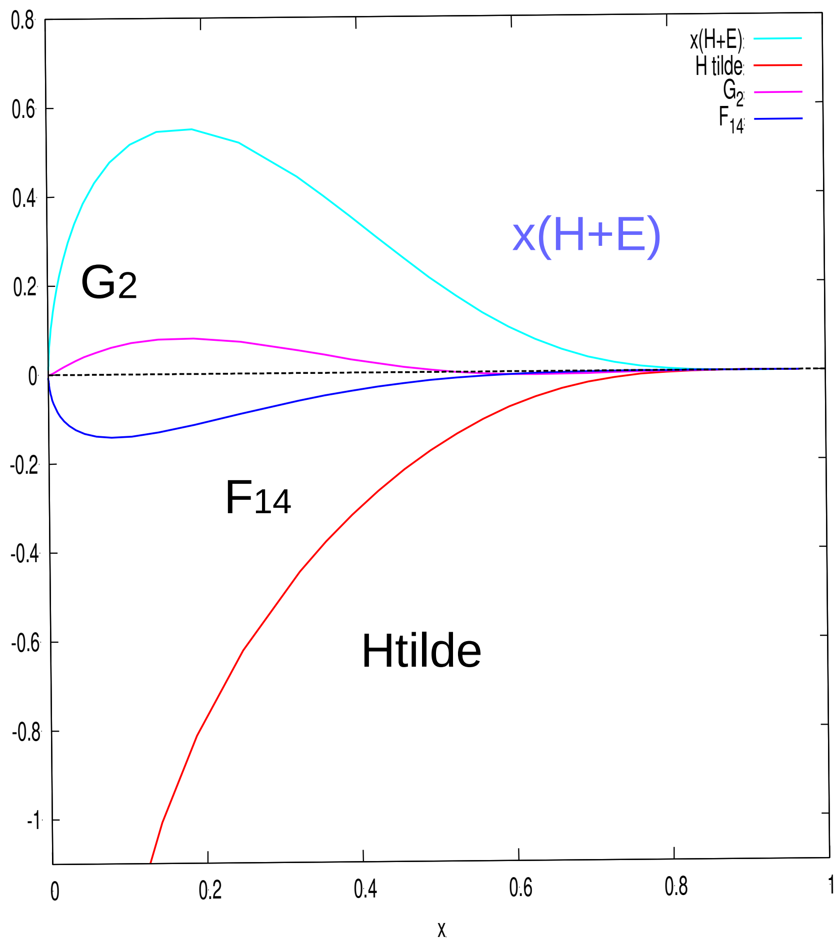
Helicity Amplitudes For G2

$$
\begin{aligned}
W^{\gamma^1}_{++} &= A^{tw3}_{+-^*,++} + A^{tw3}_{++,+-^*} - A^{tw3}_{++^*,+-} - A^{tw3}_{+-,++^*} \\
W^{\gamma^1}_{--} &= A^{tw3}_{--^*,-+} + A^{tw3}_{-+,--^*} - A^{tw3}_{-+^*,--} - A^{tw3}_{--,-+^*} \\
iW^{\gamma^2}_{++} &= -A^{tw3}_{+-^*,++} + A^{tw3}_{++,+-^*} - A^{tw3}_{++^*,+-} + A^{tw3}_{+-,++^*} \\
iW^{\gamma^2}_{--} &= -A^{tw3}_{--^*,-+} + A^{tw3}_{-+,--^*} - A^{tw3}_{-+^*,--} + A^{tw3}_{--,-+^*}
\end{aligned}
$$

$$
W^{\gamma^1}_{++} - W^{\gamma^1}_{--} - i(W^{\gamma^2}_{++} - W^{\gamma^2}_{--}) = (k_1 - ik_2)F_{27} + (\Delta_1 - i\Delta_2)F_{28}
$$

$$
-2\int \frac{d^2k_T}{(2\pi)^2}\frac{k_T.\Delta_T}{\Delta_T^2}F_{27} + F_{28} = G_2 \qquad (\Delta^+ = 0)
$$

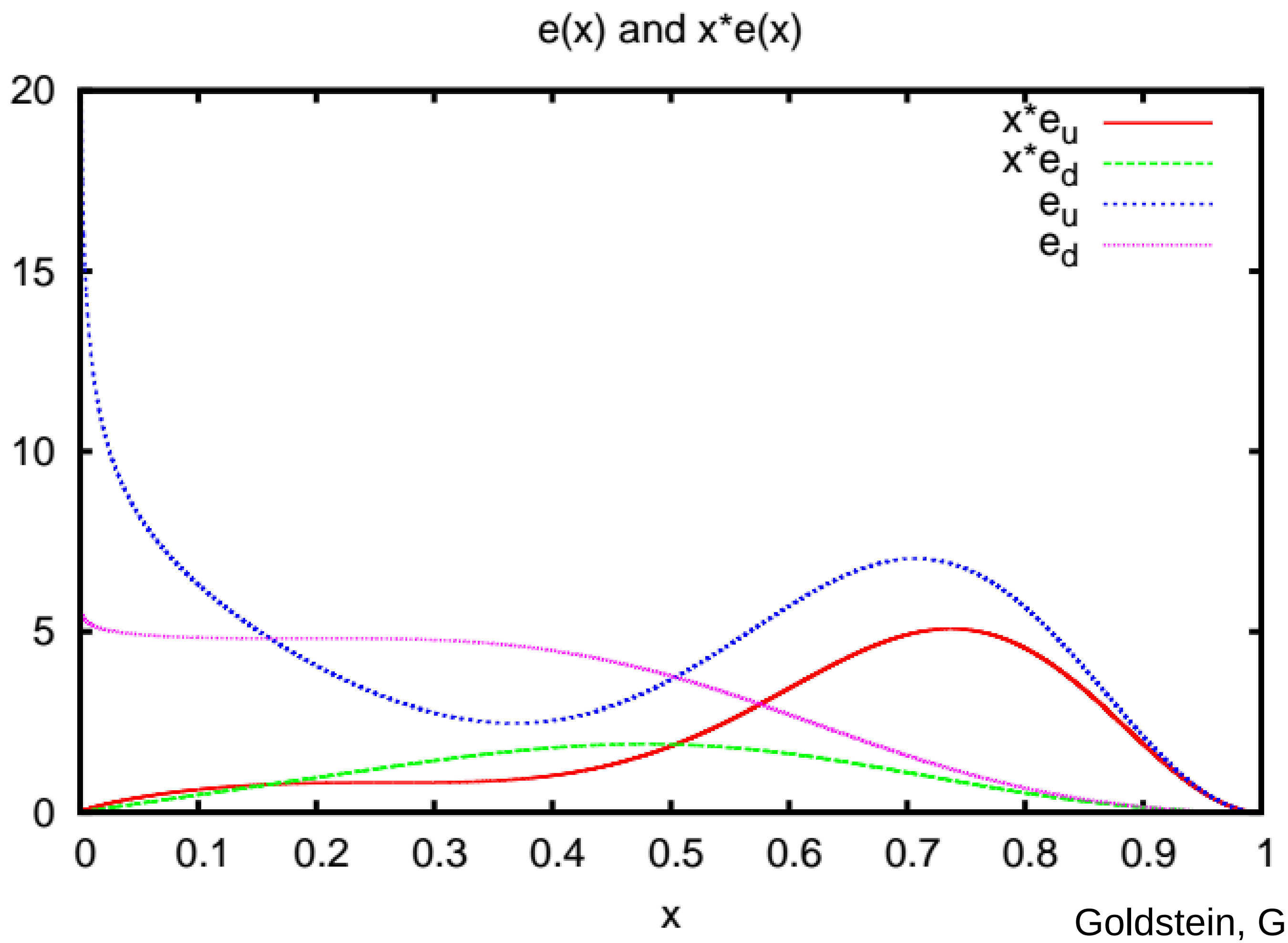
$G_2(x)$ vs x



$$-\int_{-1}^{1} dx x G_2(x,0,0) = \frac{1}{2}[\int_{-1}^{1} \tilde{H}(x,0,0) - \int_{-1}^{1} dx x (H(x,0,0) + E(x,0,0))]$$

$$L_q^{WW}(x,0,0) = x\int_x^1 \frac{dy}{y}(H_q(y,0,0) + E_q(y,0,0)) - x\int_x^1 \frac{dy}{y^2}\widetilde{H}_q(y,0,0)$$

e(x) vs x



Goldstein, Gonzalez, Liuti
PRD 84 (2011)

Using parameters from GGL.

Using Twist 3 amplitudes in the forward case

Summary and Conclusions

- GPDs, TMDs and GTMDs carry a wealth of information about partonic structure of the nucleon
- Calculation of twist 3 amplitudes allows us access to a whole new range of phenomenon
    - Partonic Orbital Angular Momentum : GPD G2
    - Color Electric and Magnetic forces : d2
    - Confinement
- Further work in separating the Wandzura Wilczek and genuine twist three (explicit gluon) contributions

$$g_2(x,Q^2) = g_2(x,Q^2)^{\mathrm{WW}} + \bar{g}_2(x,Q^2)$$

- Understand the connection between Jaffe Manohar and Ji OAM

Summary and Conclusions

- GPDs, TMDs and GTMDs carry a wealth of information about partonic structure of the nucleon
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  - Partonic Orbital Angular Momentum : GPD G_2
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Thank you!